

Energy exchange in fast optical soliton collisions as a random cascade model

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Abstract

We study the dynamics of a probe soliton propagating in an optical fiber and exchanging energy in fast collisions with a random sequence of pump solitons. The energy exchange is induced by Raman scattering or by cubic nonlinear loss/gain. We show that the equation describing the dynamics of the probe soliton's amplitude has the same form as the equation for the local space average of energy dissipation in random cascade models in turbulence. We characterize the statistics of the probe soliton's amplitude by the τ_q exponents from multifractal theory and by the Cramér function $S(x)$. We find that the n th moment of the two-time correlation function and the bit-error-rate contribution from amplitude decay exhibit power-law behavior as functions of propagation distance, where the exponents can be expressed in terms of τ_q or $S(x)$.

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The dynamics of localized patterns in the presence of noise and nonlinear effects is a rich subject that is of major importance in many fields, including solid state physics [1], turbulence [2], and optics [3]. Fiber optics communication systems, which employ optical pulses to represent information bits, provide an excellent example for systems where noise and nonlinearities have an important impact on pattern dynamics [3]. It is well established that the parameters characterizing optical pulses in fiber optics networks can exhibit non-Gaussian statistics [4, 5, 6, 7, 8, 9, 10]. Yet, since optical fiber systems are only weakly nonlinear, it was commonly believed that the statistics of optical pulse parameters is very different from the statistics encountered in strongly nonlinear systems, such as turbulence and chaotic flow, where intermittent dynamics exists. Two recent studies of pulse propagation in multichannel optical fiber transmission systems in the presence of delayed Raman response obtained results that stand in sharp contrast to this common belief [11, 12]. Taking into account the interplay between Raman-induced energy exchange in pulse collisions and bit pattern randomness it was shown that the pulse parameters exhibit intermittent dynamics in the sense that their normalized moments grow exponentially with propagation distance. Furthermore, it was found that this intermittent dynamic behavior has important practical consequences, by leading to relatively large values of the bit-error-rate (BER), which is the probability for an error at the output of the fiber line.

The studies in Refs. [11, 12] raise many intriguing questions, which are of fundamental importance in nonlinear optics, statistical physics, and chaos and turbulence theories. The main questions are: (1) Is the similarity between the fiber optics system and the turbulent one coincidental, or is it a consequence of a common underlying mechanism? (2) Is Raman scattering the only nonlinear process leading to intermittent dynamics of optical solitons? (3) Can the statistics of pulse parameters in the weakly nonlinear optical fiber system be analyzed by statistical mechanics tools that are used in the analysis of strongly nonlinear systems? (4) If the answer to (3) is yes, what predictions can be made for the main observables using these statistical mechanics tools? In this Letter we address these questions in detail. We focus attention on propagation of a probe soliton undergoing many collisions with a random sequence of pump solitons in the presence of delayed Raman response. First, we show that for certain setups of the pump soliton sequence, the dynamic equation for the probe soliton's amplitude has the same form as the equation for the local space average of energy dissipation in random cascade models in turbulence. The latter models play

a pivotal role in the analysis of large fluctuations of the local energy dissipation and in obtaining corrections to the scaling laws of the classical Kolmogorov theory of turbulence [2, 13, 14, 15, 16, 17, 18, 19, 20]. Thus, our study provides a surprising answer to question (1): the similarity between the weakly nonlinear system and the strongly nonlinear one is not coincidental. Second, we demonstrate that Raman scattering is not the only nonlinear effect leading to intermittent dynamics and that similar behavior should be observed in systems where the collision-induced energy exchange is due to cubic nonlinear loss/gain. In this sense our results are quite general, since they hold for the three main first order perturbations to the nonlinear Schrödinger (NLS) equation that lead to collision-induced energy exchange. Third, we obtain a positive answer to question (3) by showing that the statistics of the probe soliton's amplitude can be described by the τ_q exponents, which are commonly employed for analyzing multifractals and strange attractors in chaotic systems [16, 17, 18], and by relating the τ_q exponents to the Cramér function $S(x)$ via a Legendre transform. Fourth, we examine the implications of this dynamic behavior on two major observables of the probe soliton: the n th moment of the two-time equal-distance correlation function and the BER contribution from amplitude decay. We find that both observables exhibit power-law behavior as functions of propagation distance, where the exponents can be expressed in terms of τ_q or $S(x)$. We also reveal an intriguing similarity between BER dynamics and the dynamics of the d -measure in coarsening of geometrical multifractals.

It is interesting to note that several previous works studied the emergence of optical turbulence in nonlinear cavity ring resonators [21, 22, 23, 24]. In one of these works (Ref. [24]) the emergence of turbulent-like behavior was associated with the excitation of many strongly interacting spatio-temporal structures, but the phenomenon was not analyzed quantitatively. In addition, direct mathematical links between dynamics in systems described by NLS and related models and dynamics observed in models of turbulent flow have been obtained in Refs. [25, 26, 27, 28] (see also references therein). However, the propagation equations in these studies did not take into account the effects of Raman scattering, and the setups considered were different from the setups considered here. Furthermore, the statistics of the physical observables was not analyzed in terms of the random cascade model and the multifractal formalism that are employed in the current paper.

Propagation of pulses of light in an optical fiber in the presence of delayed Raman response

is described by the following perturbed NLS equation [3]

$$i\partial_z\Psi + \partial_t^2\Psi + 2|\Psi|^2\Psi = -\epsilon_R\Psi\partial_t|\Psi|^2, \quad (1)$$

where Ψ is the envelope of the electric field, z is propagation distance and t is retarded time. The term $-\epsilon_R\Psi\partial_t|\Psi|^2$ accounts for the effects of delayed Raman response and ϵ_R is the Raman coefficient [29]. When $\epsilon_R = 0$, the solution of Eq. (1) corresponding to a single soliton with frequency β is described by $\Psi_\beta(t, z) = \eta_\beta \exp(i\chi_\beta) \cosh^{-1}(x_\beta)$, where $x_\beta = \eta_\beta(t - y_\beta - 2\beta z)$, $\chi_\beta = \alpha_\beta + \beta(t - y_\beta) + (\eta_\beta^2 - \beta^2)z$, and η_β, α_β and y_β are the soliton amplitude, phase and position.

Consider a single collision between a probe soliton with frequency $\beta = 0$ and a pump soliton with frequency β . In a fast collision $|\beta| \gg 1$. Assuming in addition that $\epsilon_R \ll 1$, the main effect of delayed Raman response on the collision is an $O(\epsilon_R)$ change in the soliton amplitude [8, 10, 30, 31, 32]:

$$\Delta\eta_0 = 2\eta_0\eta_\beta\text{sgn}(\beta)\epsilon_R. \quad (2)$$

The effect of the collision in order ϵ_R/β is a frequency shift given by [8, 10, 30, 32]: $\Delta\beta_0 = -(8\eta_0^2\eta_\beta\epsilon_R)/(3|\beta|)$. This effect and effects of order ϵ_R^2 and higher can be neglected for the dynamical setups considered below. In addition, for other types of perturbations, such as those due to third order dispersion, the collision-induced changes in amplitude and frequency are of higher order in both the parameter ϵ characterizing the perturbative process and $1/|\beta|$ (see, e.g., Refs. [33, 34, 35]).

We now describe propagation of a probe soliton under many collisions with a random sequence of pump solitons. The pump solitons are located at time slot centers, and each time slot can be either occupied or empty. The occupation state of the j th time slot is described by the random variable ζ_j : $\zeta_j = 1$ with probability s if the slot is occupied and $\zeta_j = 0$ with probability $1 - s$ otherwise. It is assumed that different time slots are uncorrelated: $\langle\zeta_i\zeta_j\rangle = s^2$ if $i \neq j$. The probe soliton is initially located at $y_0(0) = 0$ and the frequencies of the pump solitons are $\beta_j = \Delta\beta > 0$. Since we look for power-law behavior of the physical observables we assume that the pump solitons are initially located at $y_{\beta j}(0) = -a^j T$, where T and $a > 1$ are constants. Therefore, the collision between the j th pump soliton and the probe soliton occurs at a distance z_j , given by: $z_j = a^j \Delta z_c^{(1)}$, where $\Delta z_c^{(1)} = T/(2\Delta\beta)$.

Using Eq. (2) and assuming that $\eta_{\beta j}(0) = \eta_\beta(0)$ for all pump solitons we find that the probe soliton's amplitude after J collisions is $\eta_0(z_J) = \eta_0(z_{J-1})[1 + 2\epsilon_R\eta_\beta(0)\zeta_J]$, leading

to $\eta_0(z_J) = \eta_0(0) \prod_{j=1}^J W_j$, where $W_j = 1 + 2\epsilon_R \eta_\beta(0) \zeta_j$. Compensation of average cross talk effects can be introduced in a straightforward manner. In this case the probe soliton's amplitude after J collisions is $\eta_0(z_J) = \eta_0(z_{J-1})[1 + 2\epsilon_R \eta_\beta(0)(\zeta_J - s)]$, resulting in

$$\eta_0(z_J) = \eta_0(0) \prod_{j=1}^J \tilde{W}_j, \quad (3)$$

where $\tilde{W}_j = 1 + 2\epsilon_R \eta_\beta(0)(\zeta_j - s)$. Notice that $\langle \tilde{W}_j \rangle = 1$, and therefore $\langle \eta_0(z_J) \rangle = 1$. Equation (3) has the same form as the equation for the local space average of energy dissipation in random cascade models in turbulence. (Compare with Ref. [2], p. 166). In this equivalence, z_J , $\Delta z_c^{(1)}$ and $\eta_0(z_J)$, play the roles of eddy size l , upper turbulence cutoff l_0 , and energy dissipation of eddies of size l , ε_l , respectively. The evolution of both η_0 and ε_l is multiplicative and dissipative. Notice that in the fiber optics system energy cascades between successive collisions, whereas in the turbulent model the cascade is from large eddies to smaller ones. Using Eq. (3) and the statistical independence of the \tilde{W}_j factors, we obtain

$$\langle \eta_0^q(z_J) \rangle = \eta_0^q(0) (z_J / \Delta z_c^{(1)})^{\log_a \langle \tilde{W}^q \rangle}. \quad (4)$$

We define the τ_q moments in a similar manner to Refs. [16, 17, 18]: $\tau_q \equiv \log_a \langle \tilde{W}^q \rangle$. For the fiber optics system:

$$\langle \tilde{W}^q \rangle = s w_1^q + (1 - s) w_2^q, \quad (5)$$

where $w_1 = 1 + 2(1 - s)\epsilon_R \eta_\beta(0)$ and $w_2 = 1 - 2s\epsilon_R \eta_\beta(0)$. The τ_q curve obtained by using Eq. (5) is plotted in Fig. 1 for $\epsilon_R = 0.03$ ($\tilde{\tau}_0 = 0.2$ ps), $a = 1.25$, $\eta_\beta(0) = 1$, and different s -values.

We emphasize that similar dynamics of the probe soliton's amplitude is expected in systems described by perturbed NLS equations, where the perturbation is due to cubic nonlinear loss/gain. For these systems the $-\epsilon_R \Psi \partial_t |\Psi|^2$ term on the right hand side of Eq. (1) is replaced by $\mp \epsilon_c |\Psi|^2 \Psi$, respectively, where ϵ_c is the cubic nonlinear loss/gain. The main effect of a fast collision in this case is an amplitude change, which is given by an equation of the form (2) with $\text{sgn}(\beta)\epsilon_R$ replaced by $\mp 2\epsilon_c/|\beta|$. Thus, the results of this Letter are quite general since they hold for Raman scattering and nonlinear loss/gain, which are the three main first order perturbations to the NLS equation that lead to energy exchange in pulse collisions.

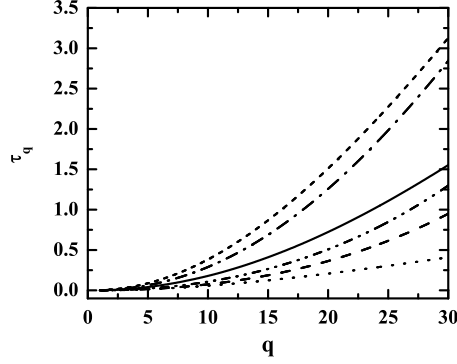


FIG. 1: The τ_q curve for the probe soliton's amplitude for $\epsilon_R = 0.03$ (pulse width = 0.2 ps), $a = 1.25$, and $\eta_{\beta}(0) = 1$. The solid, dashed and dotted lines represent the values obtained in the case where $\eta_{\beta j}(0)$ is deterministic with $s = 0.5$, $s = 0.1$, and $s = 0.9$, respectively. The dashed-dotted, dashed-dotted-dotted, and short-dashed curves correspond to τ_q in the case where $\eta_{\beta j}(0)$ is random with $s = 0.5$, $\Delta\eta_{\beta j} = 0.5$, and $\rho = 0.5$, $\rho = 0.1$, $\rho = 0.9$, respectively.

It is possible to relate the τ_q exponents to the Cramér function $S(x)$, characterizing the statistics of the pump soliton bit pattern, by a method similar to the one used for random cascade models and multifractal sets [16, 17, 18]. For this purpose we notice that the variable $m = \sum_{j=1}^J \zeta_j$ is binomially distributed with probability density function (PDF) $P(m; J) = [J!s^m(1-s)^{J-m}]/[m!(J-m)!]$. Using Stirling's formula we obtain

$$P(m; J) \simeq [-S''(x)]^{1/2} \exp[JS(x)] / (2\pi J)^{1/2}, \quad (6)$$

where $x = m/J$ and

$$S(x) = -x \ln(x/s) - (1-x) \ln[(1-x)/(1-s)]. \quad (7)$$

To obtain the relation between τ_q and $S(x)$ we notice that $a^{J\tau_q} = \langle \prod_{j=1}^J \tilde{W}_j^q \rangle$, and consequently: $a^{J\tau_q} = \langle \exp \left\{ q \sum_{j=1}^J \ln [1 + 2\epsilon_R(\zeta_j - s)] \right\} \rangle$. Expressing the sum in the exponent's argument in terms of x , w_1 and w_2 and using the large-deviations theorem we obtain

$$a^{J\tau_q} \simeq \int dx \left[\frac{-JS''(x)}{2\pi} \right]^{1/2} \exp \{ qJ [x \ln(w_1/w_2) + \ln(w_2)] + JS(x) \}. \quad (8)$$

Employing a saddle point approximation we obtain the Legendre transform relating τ_q and $S(x)$:

$$\tau_q = \{ q [x \ln(w_1/w_2) + \ln(w_2)] + S(x) \} / \ln a, \quad (9)$$

where $x(q)$ is determined by the conditions $q \ln(w_1/w_2) = -S'(x)$, and $S''(x) < 0$.

Since $\epsilon_R \ll 1$ the \tilde{W}_j factors in the fiber optics system are only slightly different from 1. It is therefore interesting to look for ways to enhance the randomness of the collision-induced energy exchange. Here we describe one disorder enhancement mechanism, which also leads to a natural generalization of the model described by Eq. (3). Consider a setup in which the amplitude of the j th pump soliton is given by $\eta_{\beta j}(0) = \eta_\beta(0) \pm \Delta\eta_\beta$ with probabilities ρ and $1 - \rho$, respectively. A straightforward analysis shows that the η_0 dynamics is still described by Eq. (3). However, the \tilde{W}_j factors can now attain three values: $w_1 = 1 + 2\epsilon_R[(1-s)\eta_\beta(0) + (1-2\rho)s\Delta\eta_\beta + \Delta\eta_\beta]$ with probability ρs , $w_2 = 1 + 2\epsilon_R[(1-s)\eta_\beta(0) + (1-2\rho)s\Delta\eta_\beta - \Delta\eta_\beta]$ with probability $(1-\rho)s$, and $w_3 = 1 + 2\epsilon_R[-s\eta_\beta(0) + (1-2\rho)s\Delta\eta_\beta]$ with probability $1-s$. The average of \tilde{W}^q is

$$\langle \tilde{W}^q \rangle = \rho s w_1^q + (1-\rho)s w_2^q + (1-s)w_3^q. \quad (10)$$

The τ_q curve for this model is shown in Fig. 1. One can see that τ_q attains larger values when $\eta_{\beta j}(0)$ is random for $s = 0.5$, $\rho = 0.5$, and $\rho = 0.9$, compared with the case where $\eta_{\beta j}(0)$ is deterministic and $s = 0.5$. Thus, the cumulative effect of energy exchange in the collisions is indeed enhanced by randomness of $\eta_{\beta j}(0)$. To obtain the Legendre transform between τ_q and $S(x)$, we denote by $m_i, i = 1, 2, 3$, the number of occurrences of pump solitons with $\tilde{W}_j = w_i$. The PDF of m_1 and m_2 is trinomial: $P(m_1, m_2; J) = \{J! (\rho s)^{m_1} [(1-\rho)s]^{m_2} (1-s)^{J-m_1-m_2}\} / [m_1! m_2! (J-m_1-m_2)!]$. Using Stirling's formula we arrive at

$$P(m_1, m_2; J) \simeq \frac{J^{1/2} \exp[J\tilde{S}(x_1, x_2)]}{2\pi [m_1 m_2 (J-m_1-m_2)]^{1/2}}, \quad (11)$$

where $x_{1,2} = m_{1,2}/J$, and

$$\begin{aligned} \tilde{S}(x_1, x_2) = & -x_1 \ln[x_1/(\rho s)] - x_2 \ln\{x_2/[(1-\rho)s]\} \\ & -(1-x_1-x_2) \ln[(1-x_1-x_2)/(1-s)]. \end{aligned} \quad (12)$$

By employing the large-deviations theorem and a saddle point calculation we obtain the following generalization of the Legendre transform given by Eq. (9):

$$\begin{aligned} \tau_q = & \{q [x_1 \ln(w_1/w_3) + x_2 \ln(w_2/w_3) + \ln(w_3)] \\ & + \tilde{S}(x_1, x_2)\} / \ln a, \end{aligned} \quad (13)$$

where $x_1(q)$ and $x_2(q)$ are determined by

$$q \ln(w_i/w_3) = -\frac{\partial \tilde{S}}{\partial x_i} \Big|_{x_i=x_i(q)}, \quad i = 1, 2, \quad (14)$$

$\partial^2 \tilde{S} / \partial^2 x_1 < 0$, and $(\partial^2 \tilde{S} / \partial^2 x_1)(\partial^2 \tilde{S} / \partial^2 x_2) - (\partial^2 \tilde{S} / \partial x_1 \partial x_2)^2 > 0$. Since the exact expression for $\langle \tilde{W}^q \rangle$ [Eq. (10)] can be retrieved by using Eqs. (12)-(14) we conclude that the τ_q exponents give a correct characterization of the statistics of η_0 . In addition, Eqs. (12)-(14) can be further generalized for the case where the \tilde{W}_j factors attain any finite number of values.

We now discuss implications of the dynamics of the probe soliton's amplitude. We start by considering the n th moment of the two-time equal-distance correlation:

$$C_{01}^{(n)} = \langle (\eta_{00}\eta_{01})^n \rangle / (\langle \eta_{00} \rangle^n \langle \eta_{01} \rangle^n). \quad (15)$$

The function $C_{01}^{(n)}$ measures correlation between the amplitudes η_{00} and η_{01} of two probe solitons, whose initial positions are $y_{00}(0) = 0$ and $y_{01}(0) = AT$, respectively, where A is a constant. We remark that high moments of velocity correlation function along a given direction play an important role in turbulence theory [2]. From Eq. (3) it follows that $\langle [\eta_{00}(z_J)\eta_{01}(z_J)]^n \rangle = \eta_{00}^n(0)\eta_{01}^n(0)\langle \prod_{j=1}^J \tilde{W}_{j0}^n \prod_{j=1}^{j_{max}} \tilde{W}_{j1}^n \rangle$, where j_{max} is the number of collisions experienced by the 01 probe soliton, and $\tilde{W}_{j1} = \tilde{W}_{j0}$ for $j = 1, \dots, j_{max}$. Using this relation, the statistical independence of the \tilde{W}_{j0} factors and the definition of the τ_q exponents, we obtain

$$C_{01}^{(n)}(z_J) \simeq \left[\frac{z_J - z_{11}}{\Delta z_c^{(1)}} \right]^{\tau_{2n}} \left[\frac{z_J}{z_J - z_{11}} \right]^{\tau_n}, \quad (16)$$

where $z_{11} = A\Delta z_c^{(1)}$. Thus, the n th moment of the two-time correlation function is a product of power-laws of two different scaled distances.

One of the most important quantities characterizing the performance of fiber optics systems is the BER. Since the BER is often determined by the tail of the PDF of the pulse parameters one can expect that it would be closely related to the Cramér function. We show that this is indeed the case for the system described here. We consider the setup where the amplitudes of the pump solitons are deterministic [$\eta_{\beta j}(0) = \eta_{\beta}(0)$] and focus attention on the contribution to the probe soliton's BER due to amplitude decay, BER_{η} . This contribution is defined by: $\text{BER}_{\eta} \equiv \int_0^{\eta_{th}} d\eta_0 F(\eta_0)$, where $F(\eta_0)$ is the amplitude PDF and η_{th} is the

threshold for an error. Using $\eta_0(z_J) = \eta_0(0)w_1^m w_2^{J-m}$ and Eq. (6) we obtain

$$\text{BER}_\eta \simeq \sum_{m=0}^{m_{th}} \frac{J^{1/2} \exp[JS(m/J)]}{[2\pi m(J-m)]^{1/2}}, \quad (17)$$

where m_{th} is the solution of $\eta_0(m) = \eta_{th}$. Since $S(x)$ is an increasing function of x the main contribution to the sum on the right hand side of Eq. (17) comes from a close neighborhood of m_{th} . Taking into account the two leading terms we arrive at:

$$\text{BER}_\eta \simeq \left[\frac{-S''(x)}{2\pi J} \right]^{1/2} \left[\frac{z_J}{\Delta z_c^{(1)}} \right]^{\frac{S(x_{th})}{\ln a}} \left[1 + a^{-\frac{S'(x_{th})}{\ln a}} \right], \quad (18)$$

where $x_{th} = m_{th}/J$. Thus, in the leading order BER_η grows like a power law with propagation distance, where the exponent is the value of the Cramér function at the error threshold. The ϵ_R -dependence of BER_η in the range $0.0075 \leq \epsilon_R \leq 0.012$ ($0.5\text{ps} \leq \tilde{\tau}_0 \leq 0.8\text{ps}$) is shown in Fig. 2 for $\eta_0(0) = 1$, $\eta_\beta(0) = 2$, $\eta_{th} = 0.7$ and $J = 27$. For the choice $a = 1.25$, $T = 5$, $\Delta\beta = 40$, and $\beta_2 = -4\text{ps}^2/\text{km}$, for example, we obtain $7.96\text{THz} < \Delta\nu < 12.74\text{THz}$ for the frequency difference, and $1.07\text{km} < X_{27} < 2.75\text{km}$ for the propagation distance. It is seen that both Eq. (18) and the first term on the right hand side of Eq. (18) are good approximations to the exact result over a wide range of BER_η values. It is interesting that the dynamic behavior of BER_η is very similar to the behavior of the d -measure of geometrical multifractals during the late stage of coarsening (compare Eq. (17) with Eq. (9) in Ref. [36]).

In many cases the dynamics of the other three soliton parameters is coupled to the amplitude dynamics and as a result, these parameters can be strongly influenced by amplitude fluctuations. For the system considered here the most important effect is due to the Raman-induced self frequency shift, which is given by [37]: $\beta_0(z) = -(8\epsilon_R/15) \int_0^z dz' \eta_0^4(z')$. This frequency shift leads to a position shift:

$$y_0(z) = -\frac{16\epsilon_R}{15} \int_0^z dz' \int_0^{z'} dz'' \eta_0^4(z'') \quad (19)$$

that can give significant contribution to the probe soliton's total BER. It is therefore important to evaluate the impact of this process on the total BER in comparison with the contribution BER_η coming solely from amplitude decay. Since we do not have an analytic expression for the PDF of y we carry out Monte Carlo simulations with Eqs. (3) and (19). We define the relative position shift $\tilde{y}_0 = y_0 - \langle y_0 \rangle$, where the average $\langle y_0 \rangle$ is assumed to be

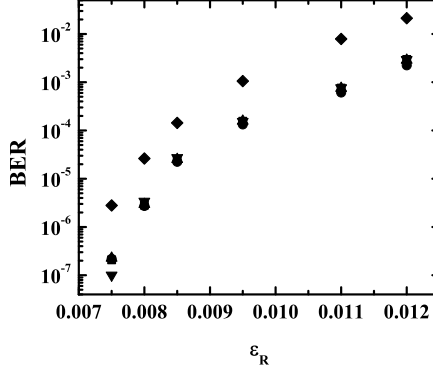


FIG. 2: Total BER and BER contribution due to pulse decay, BER_η , vs the Raman coefficient ϵ_R for $\eta_0(0) = 1$, $\eta_\beta(0) = 2$, $a = 1.25$, $T = 5$, and $\beta_2 = -4\text{ps}^2/\text{km}$. The squares, up triangles, and circles stand for the exact result for BER_η , the approximate expression given by Eq. (18), and the first term on the right hand side of Eq. (18), respectively. The down triangles and diamonds represent the total BER for $\Delta\beta = 40$ and $\Delta\beta = 20$, respectively.

compensated by filters. For each realization of the W_j factors we compute the total energy at the detector at distance z_J :

$$I(z_J) = \eta_0(z_J) \{ \tanh[\eta_0(z_J)(T/2 - \tilde{y}_0(z_J))] + \tanh[\eta_0(z_J)(T/2 + \tilde{y}_0(z_J))] \}. \quad (20)$$

An occupied time slot is considered to be in error, if $I(z_J) < I_{th} = 2\eta_{th} \tanh(\eta_{th}T/2)$. We use the same parameter values as described in the previous paragraph, but with two different values of $\Delta\beta$: $\Delta\beta = 40$ and $\Delta\beta = 20$ [38]. The total BER obtained in the simulations is shown in Fig. 2. It is seen that for $\Delta\beta = 40$ the total BER is very close to BER_η , that is, error generation is dominated by amplitude decay. In contrast, for $\Delta\beta = 20$, error generation is dominated by the Raman-induced position shift, and as a result the total BER is much larger than BER_η . These results can be explained by noting that the smaller inter-collision distances for $\Delta\beta = 40$ lead to a smaller total propagation distance, and consequently, the position shift is relatively small. For $\Delta\beta = 20$, the inter-collision distances and the total propagation distance are large, resulting in relatively large position shifts.

In summary, we studied the amplitude dynamics of a probe NLS soliton, exchanging energy in fast collisions with a random sequence of pump solitons. We showed that the equation for the probe soliton's amplitude has the same form as the equation for the local space average of energy dissipation in random cascade models in turbulence. We found

that the n th moment of the two-time correlation function and the BER contribution from amplitude decay exhibit power-law behavior as functions of propagation distance, where the exponents can be expressed in terms of the τ_q exponents or the Cramér function. Thus, our study provides a surprising and very useful perspective on the relation between disorder effects on weakly nonlinear systems described by perturbed NLS equations, and strongly nonlinear systems, such as turbulent flow.

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